Solar Neutrino Solutions in Non-Abelian Flavor Symmetry

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We have studied the large mixing angle MSW solution for the solar neutrinos in the non-abelian flavor symmetry. We predict the MNS mixing matrix taking account of the symmetry breakings.

1 LMA-MSW Solution in Non-Abelian Flavor Symmetry

Recent data in S-Kam favor the large mixing angle MSW (LMA-MSW) solution. How does one get the LMA-MSW solution as well as the maximal mixing of the atmospheric neutrinos in theory? It is not easy to reproduce the nearly bi-maximal mixings with LMA-MSW solution in GUT models[1, 2, 3].

The non-abelian flavor symmetry $S_{3L} \times S_{3R}$ or $O_{3L} \times O_{3R}$ leads to the LMA-MSW solution naturally[4, 5]. The mass matrices are

$$M_E \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \ M_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The orthogonal matrix diagonalizes M_E is

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}. \qquad (1)$$

The MNS mixing matrix is given as $U_\nu \simeq F^T$, and so we predict $\sin^2 2\theta_\odot = 1$ and $\sin^2 2\theta_{\rm atm} = 8/9$ in the symmetric limit.

In this talk, we discuss masses and flavor mixings of quarks/leptons in the non-abelian flavor symmetry with the SU(5) GUT[4]. We consider $O(3)_{\mathbf{5}^*} \times O(3)_{\mathbf{10}} \times Z_6$ symmetry. Our scenario for fermion masses is

- Neutrinos have degenerate masses.
- Quarks/charged-leptons are massless.
- Symmetry breakings give Δm^2 and other fermion masses.

2 $\mathbf{O(3)_{5^*} \times O(3)_{10} \times Z_6}$ Symmetry

Quarks and leptons belong to $\mathbf{5}^*$ and $\mathbf{10}$ of the $SU(5)$ GUT and $\mathbf{3}$ of the $O(3)$ symmetry. Higgs H ($\overline{H}$) belong to $\mathbf{5}$ ($\mathbf{5}^*$) of the $SU(5)$ and $\mathbf{1}$ of the $O(3)$. Then, neutrinos have the $O(3)_{\mathbf{5}^*} \times O(3)_{\mathbf{10}}$ invariant mass term

$$\frac{<H>^2}{\Lambda}\nu_L\nu_L \ . \qquad (2)$$

The Z_6 symmetry forbids $\psi_{\mathbf{10}}(\mathbf{3})\psi_{\mathbf{10}}(\mathbf{3})H$, which gives degenerate up-quark masses[4].

The flavor symmetry is broken explicitly by $\Sigma^{(i)}_{\mathbf{5}^*}(\mathbf{5},\mathbf{1})$, $\Sigma^{(i)}_{\mathbf{10}}(\mathbf{1},\mathbf{5})$ $(i=1,2)$, which transform as the symmetric traceless tensor $\mathbf{5}$'s of $O(3)$. Dimentionless breaking parameters are given as

$$\sigma^{(1)}_{\mathbf{10},\ \mathbf{5}^*} \equiv \frac{\Sigma^{(1)}_{\mathbf{10},\ \mathbf{5}^*}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \delta_{\mathbf{10},\ \mathbf{5}^*},$$

$$\sigma^{(2)}_{\mathbf{10},\ \mathbf{5}^*} \equiv \frac{\Sigma^{(2)}_{\mathbf{10},\ \mathbf{5}^*}}{M_f} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \epsilon_{\mathbf{10},\ \mathbf{5}^*}.$$

Neutrinos get Majorana masses from a superpotential

$$W = \frac{H^2}{\Lambda}\ell(\mathbf{1} + \alpha_i \sigma^{(i)}_{\mathbf{5}^*})\ell \ , \qquad (3)$$

which yields a diagonal neutrino mass matrix. In order to get the charged lepton masses, we introduce $O(3)_{\mathbf{5}^*}$-triplet $\phi_{\mathbf{5}^*}(\mathbf{3},\mathbf{1})$ and $O(3)_{\mathbf{10}}$-triplet $\phi_{\mathbf{10}}(\mathbf{1},\mathbf{3})$. These VEV's are determined by the superpotential

$$\begin{aligned} W &= Z_{\mathbf{5}^*}(\phi^2_{\mathbf{5}^*} - 3v^2_{\mathbf{5}^*}) + Z_{\mathbf{10}}(\phi^2_{\mathbf{10}} - 3v^2_{\mathbf{10}}) \\ &+ X_{\mathbf{5}^*}(a_{(i)}\phi_{\mathbf{5}^*}\sigma^{(i)}_{\mathbf{5}^*}\phi_{\mathbf{5}^*}) + X_{\mathbf{10}}(a'_{(i)}\phi_{\mathbf{10}}\sigma^{(i)}_{\mathbf{10}}\phi_{\mathbf{10}}) \\ &+ Y_{\mathbf{5}^*}(b_{(i)}\phi_{\mathbf{5}^*}\sigma^{(i)}_{\mathbf{5}^*}\phi_{\mathbf{5}^*}) + Y_{\mathbf{10}}(b'_{(i)}\phi_{\mathbf{10}}\sigma^{(i)}_{\mathbf{10}}\phi_{\mathbf{10}}) \end{aligned}$$

where $Z_{\mathbf{10},\ \mathbf{5}^*}$, $X_{\mathbf{10},\ \mathbf{5}^*}$, $Y_{\mathbf{10},\ \mathbf{5}^*}$ are all singlets of $O(3)_{\mathbf{5}^*} \times O(3)_{\mathbf{10}}$. Minimizing the potential, we get

$$< \phi_{\mathbf{5}^*} > \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_{\mathbf{5}^*}, \ < \phi_{\mathbf{10}} > \equiv \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} v_{\mathbf{10}}.$$

Masses of charged leptons arise from a superpotential

$$W = \frac{\kappa_E}{M_f^2} (\overline{e} \phi_{\mathbf{10}})(\phi_{\mathbf{5}^*} \ell) \overline{H}, \tag{4}$$

which is the realization of "Democratic Mass Matrix",

$$M_E \propto \left(\frac{v_{\mathbf{5}^*} v_{\mathbf{10}}}{M_f^2} \right) \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{5}$$

Adding the superpotential containing the flavor symmetry breaking parameters $\sigma^{(i)}_{\mathbf{5}^*,\ \mathbf{10}}$, we get the charged lepton mass matrix:

$$M_E^H \equiv F^T M_E F = \kappa_E \left(\frac{v_{\mathbf{5}^*} v_{\mathbf{10}}}{M_f^2} \right) < \overline{H} > \\ \times \begin{pmatrix} \epsilon_{\mathbf{5}^*} \epsilon_{\mathbf{10}} & \epsilon_{\mathbf{10}} \delta_{\mathbf{5}^*} & \epsilon_{\mathbf{10}} \\ \epsilon_{\mathbf{5}^*} \delta_{\mathbf{10}} & \delta_{\mathbf{5}^*} \delta_{\mathbf{10}} & \delta_{\mathbf{10}} \\ \epsilon_{\mathbf{5}^*} & \delta_{\mathbf{5}^*} & 3 \end{pmatrix}, \tag{6}$$

in which order one coefficients are omitted. The mass ratios are given as

$$\frac{m_\mu}{m_\tau} \simeq \mathcal{O}(\delta_{\mathbf{5}^*} \delta_{\mathbf{10}}), \quad \frac{m_e}{m_\tau} \simeq \mathcal{O}(\epsilon_{\mathbf{5}^*} \epsilon_{\mathbf{10}}).$$

The quark/lepton masses and mixings fix

$$\delta_{\mathbf{10}} \simeq \lambda^2, \ \epsilon_{\mathbf{10}} \simeq \lambda^3 \sim \lambda^4, \ \delta_{\mathbf{5}^*} \simeq \lambda, \ \epsilon_{\mathbf{5}^*} \simeq \lambda^2.$$

3 Neutrino Masses and Mixings

Neutrino masses are given as

$$\begin{aligned} m_1 \simeq & \quad c_\mu (1 + \alpha_1 \delta_{\mathbf{5}^*} + \alpha_2 \epsilon_{\mathbf{5}^*}), \\ m_2 \simeq & \quad c_\mu (1 + \alpha_1 \delta_{\mathbf{5}^*} - \alpha_2 \epsilon_{\mathbf{5}^*}), \\ m_3 \simeq & \quad c_\mu (1 - 2\alpha_1 \delta_{\mathbf{5}^*}), \quad c_\mu = \frac{< H >^2}{\Lambda} \end{aligned}$$

which leads to (with $\delta_{\mathbf{5}^*} \simeq \lambda, \quad \epsilon_{\mathbf{5}^*} \simeq \lambda^2$)

$$\left| \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \right| = \quad \frac{2}{3} \frac{\alpha_2 \epsilon_{\mathbf{5}^*}}{\alpha_1 \delta_{\mathbf{5}^*}} \frac{1 + \alpha_2 \epsilon_{\mathbf{5}^*}}{1 - \frac{1}{2} \alpha_1 \delta_{\mathbf{5}^*}} \simeq \lambda^2 \sim \lambda.$$

Putting $\Delta m_{32}^2 = 3 \times 10^{-3} \mathrm{eV}^2$, we predict $\Delta m_{21}^2 \simeq$ (factor) $\times\ 10^{-4} \mathrm{eV}^2$, which is consistent with the LMA-MSW solution. Flavor mixings come from the charge lepton mass matrix since the neutrino one is diagonal. The charged lepton mass matrix is diagonalized by $V_R^\dagger M_E^H V_L$, in which

$$V_L^\dagger \simeq \begin{pmatrix} 1 & \lambda & \lambda^2 \\ -\lambda & 1 & \lambda \\ -\lambda^2 & -\lambda & 1 \end{pmatrix}. \tag{7}$$

The neutrino mixing matrix is given by $V_L^\dagger F^T$. We predict

$$\begin{aligned} \sin^2 2\theta_\odot &= (1 - \frac{4}{3}\lambda^2)^2 \simeq 0.87 \\ \sin^2 2\theta_{\mathrm{atm}} &= \frac{8}{9}(1 - \lambda^2)(1 + \frac{1}{\sqrt{2}}\lambda - 2\lambda^2)^2 \\ &\simeq 0.95 \\ |U_{e3}| &= \frac{2}{\sqrt{6}} \lambda (1 - \frac{1}{\sqrt{2}}\lambda) \simeq 0.14. \end{aligned} \tag{8}$$

4 Summary

It is remarked that:

- The solar neutrino mixing $\sin^2 2\theta_\odot$ deviates from the maximal mixing (~ 0.87).
- The atmospheric neutrino mixing $\sin^2 2\theta_{\mathrm{atm}}$ deviates from 8/9 depending phase of λ.
- $\mathbf{U_{e3}}$ is near to the experimental bound of CHOOZ ($\leq \mathbf{0.16}$) .

Neutrino masses are degenerated within a factor 2. For example, we get $m_1 \simeq 0.030$eV, $m_2 \simeq 0.033$eV, $m_3 \simeq 0.058$eV, which is consistent with $\beta\beta_{0\nu}$ decay bound.

References

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